# AQA Maths Pure Core 4

## Past Paper Pack

## 2006-2013

General Certificate of Education
January 2006
Advanced Level Examination

**MATHEMATICS** **MPC4**
**Unit Pure Core 4**

Wednesday 25 January 2006 9.00 am to 10.30 am

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|---|
| **For this paper you must have:**<br>• an 8-page answer book<br>• the **blue** AQA booklet of formulae and statistical tables<br>You may use a graphics calculator. |

Time allowed: 1 hour 30 minutes

**Instructions**
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.

**Information**
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

**1** (a) The polynomial $f(x)$ is defined by $f(x) = 3x^3 + 2x^2 - 7x + 2$.

(i) Find $f(1)$. *(1 mark)*

(ii) Show that $f(-2) = 0$. *(1 mark)*

(iii) Hence, or otherwise, show that

$$\frac{(x-1)(x+2)}{3x^3 + 2x^2 - 7x + 2} = \frac{1}{ax + b}$$

where $a$ and $b$ are integers. *(3 marks)*

(b) The polynomial $g(x)$ is defined by $g(x) = 3x^3 + 2x^2 - 7x + d$.

When $g(x)$ is divided by $(3x - 1)$, the remainder is 2. Find the value of $d$. *(3 marks)*

**2** A curve is defined by the parametric equations

$$x = 3 - 4t \qquad y = 1 + \frac{2}{t}$$

(a) Find $\frac{dy}{dx}$ in terms of $t$. *(4 marks)*

(b) Find the equation of the tangent to the curve at the point where $t = 2$, giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers. *(4 marks)*

(c) Verify that the cartesian equation of the curve can be written as

$$(x - 3)(y - 1) + 8 = 0$$

*(3 marks)*

**3** It is given that $3\cos\theta - 2\sin\theta = R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$.

(a) Find the value of $R$. *(1 mark)*

(b) Show that $\alpha \approx 33.7°$. *(2 marks)*

(c) Hence write down the maximum value of $3\cos\theta - 2\sin\theta$ and find a **positive** value of $\theta$ at which this maximum value occurs. *(3 marks)*

**4** On 1 January 1900, a sculpture was valued at £80.

When the sculpture was sold on 1 January 1956, its value was £5000.

The value, £$V$, of the sculpture is modelled by the formula $V = Ak^t$, where $t$ is the time in years since 1 January 1900 and $A$ and $k$ are constants.

(a) Write down the value of $A$. *(1 mark)*

(b) Show that $k \approx 1.07664$. *(3 marks)*

(c) Use this model to:

(i) show that the value of the sculpture on 1 January 2006 will be greater than £200 000; *(2 marks)*

(ii) find the year in which the value of the sculpture will first exceed £800 000. *(3 marks)*

**5** (a) (i) Obtain the binomial expansion of $(1-x)^{-1}$ up to and including the term in $x^2$. *(2 marks)*

(ii) Hence, or otherwise, show that

$$\frac{1}{3-2x} \approx \frac{1}{3} + \frac{2}{9}x + \frac{4}{27}x^2$$

for small values of $x$. *(3 marks)*

(b) Obtain the binomial expansion of $\dfrac{1}{(1-x)^2}$ up to and including the term in $x^2$. *(2 marks)*

(c) Given that $\dfrac{2x^2-3}{(3-2x)(1-x)^2}$ can be written in the form $\dfrac{A}{(3-2x)} + \dfrac{B}{(1-x)} + \dfrac{C}{(1-x)^2}$,

find the values of $A$, $B$ and $C$. *(5 marks)*

(d) Hence find the binomial expansion of $\dfrac{2x^2-3}{(3-2x)(1-x)^2}$ up to and including the term in $x^2$. *(3 marks)*

**Turn over for the next question**

**6** (a) Express $\cos 2x$ in the form $a\cos^2 x + b$, where $a$ and $b$ are constants. *(2 marks)*

(b) Hence show that $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx = \frac{\pi}{a}$, where $a$ is an integer. *(5 marks)*

**7** The quadrilateral $ABCD$ has vertices $A\,(2, 1, 3)$, $B\,(6, 5, 3)$, $C\,(6, 1, -1)$ and $D\,(2, -3, -1)$.

The line $l_1$ has vector equation $\mathbf{r} = \begin{bmatrix} 6 \\ 1 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

(a) (i) Find the vector $\overrightarrow{AB}$. *(2 marks)*

(ii) Show that the line $AB$ is parallel to $l_1$. *(1 mark)*

(iii) Verify that $D$ lies on $l_1$. *(2 marks)*

(b) The line $l_2$ passes through $D\,(2, -3, -1)$ and $M\,(4, 1, 1)$.

(i) Find the vector equation of $l_2$. *(2 marks)*

(ii) Find the angle between $l_2$ and $AC$. *(3 marks)*

**8** (a) Solve the differential equation

$$\frac{dx}{dt} = -2(x-6)^{\frac{1}{2}}$$

to find $t$ in terms of $x$, given that $x = 70$ when $t = 0$. *(6 marks)*

(b) Liquid fuel is stored in a tank. At time $t$ minutes, the depth of fuel in the tank is $x$ cm. Initially there is a depth of 70 cm of fuel in the tank. There is a tap 6 cm above the bottom of the tank. The flow of fuel out of the tank is modelled by the differential equation

$$\frac{dx}{dt} = -2(x-6)^{\frac{1}{2}}$$

(i) Explain what happens when $x = 6$. *(1 mark)*

(ii) Find how long it will take for the depth of fuel to fall from 70 cm to 22 cm. *(2 marks)*

**END OF QUESTIONS**

General Certificate of Education
June 2006
Advanced Level Examination

**MATHEMATICS** **MPC4**
**Unit Pure Core 4**

Thursday 15 June 2006 1.30 pm to 3.00 pm

**For this paper you must have:**
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**
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- Show all necessary working; otherwise marks for method may be lost.

**Information**
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- The marks for questions are shown in brackets.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

**1** (a) The polynomial $\mathrm{p}(x)$ is defined by $\mathrm{p}(x) = 6x^3 - 19x^2 + 9x + 10$.

(i) Find $\mathrm{p}(2)$. *(1 mark)*

(ii) Use the Factor Theorem to show that $(2x + 1)$ is a factor of $\mathrm{p}(x)$. *(3 marks)*

(iii) Write $\mathrm{p}(x)$ as the product of three linear factors. *(2 marks)*

(b) Hence simplify $\dfrac{3x^2 - 6x}{6x^3 - 19x^2 + 9x + 10}$. *(2 marks)*

**2** (a) Obtain the binomial expansion of $(1 - x)^{-3}$ up to and including the term in $x^2$. *(2 marks)*

(b) Hence obtain the binomial expansion of $\left(1 - \dfrac{5}{2}x\right)^{-3}$ up to and including the term in $x^2$. *(2 marks)*

(c) Find the range of values of $x$ for which the binomial expansion of $\left(1 - \dfrac{5}{2}x\right)^{-3}$ would be valid. *(2 marks)*

(d) Given that $x$ is small, show that $\left(\dfrac{4}{2 - 5x}\right)^3 \approx a + bx + cx^2$, where $a$, $b$ and $c$ are integers. *(2 marks)*

**3** (a) Given that $\dfrac{9x^2 - 6x + 5}{(3x - 1)(x - 1)}$ can be written in the form $3 + \dfrac{A}{3x - 1} + \dfrac{B}{x - 1}$, where $A$ and $B$ are integers, find the values of $A$ and $B$. *(4 marks)*

(b) Hence, or otherwise, find $\displaystyle\int \frac{9x^2 - 6x + 5}{(3x - 1)(x - 1)}\,\mathrm{d}x$. *(4 marks)*

**4** (a) (i) Express $\sin 2x$ in terms of $\sin x$ and $\cos x$. *(1 mark)*

(ii) Express $\cos 2x$ in terms of $\cos x$. *(1 mark)*

(b) Show that

$$\sin 2x - \tan x = \tan x \cos 2x$$

for all values of $x$. *(3 marks)*

(c) Solve the equation $\sin 2x - \tan x = 0$, giving all solutions in degrees in the interval $0° < x < 360°$. *(4 marks)*

**5** A curve is defined by the equation

$$y^2 - xy + 3x^2 - 5 = 0$$

(a) Find the $y$-coordinates of the two points on the curve where $x = 1$. *(3 marks)*

(b) (i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{y - 6x}{2y - x}$. *(6 marks)*

(ii) Find the gradient of the curve at each of the points where $x = 1$. *(2 marks)*

(iii) Show that, at the two stationary points on the curve, $33x^2 - 5 = 0$. *(3 marks)*

**6** The points $A$ and $B$ have coordinates $(2, 4, 1)$ and $(3, 2, -1)$ respectively. The point $C$ is such that $\overrightarrow{OC} = 2\overrightarrow{OB}$, where $O$ is the origin.

(a) Find the vectors:

(i) $\overrightarrow{OC}$; *(1 mark)*

(ii) $\overrightarrow{AB}$. *(2 marks)*

(b) (i) Show that the distance between the points $A$ and $C$ is 5. *(2 marks)*

(ii) Find the size of angle $BAC$, giving your answer to the nearest degree. *(4 marks)*

(c) The point $P(\alpha, \beta, \gamma)$ is such that $BP$ is perpendicular to $AC$.

Show that $4\alpha - 3\gamma = 15$. *(3 marks)*

**Turn over for the next question**

**7** Solve the differential equation

$$\frac{dy}{dx} = 6xy^2$$

given that $y = 1$ when $x = 2$. Give your answer in the form $y = f(x)$. *(6 marks)*

**8** A disease is spreading through a colony of rabbits. There are 5000 rabbits in the colony. At time $t$ hours, $x$ is the number of rabbits infected. The rate of increase of the number of rabbits infected is proportional to the product of the number of rabbits infected and the number not yet infected.

(a) (i) Formulate a differential equation for $\frac{dx}{dt}$ in terms of the variables $x$ and $t$ and a constant of proportionality $k$. *(2 marks)*

(ii) Initially, 1000 rabbits are infected and the disease is spreading at a rate of 200 rabbits per hour. Find the value of the constant $k$.

(You are **not** required to solve your differential equation.) *(2 marks)*

(b) The solution of the differential equation in this model is

$$t = 4\ln\left(\frac{4x}{5000 - x}\right)$$

(i) Find the time after which 2500 rabbits will be infected, giving your answer in hours to one decimal place. *(2 marks)*

(ii) Find, according to this model, the number of rabbits infected after 30 hours. *(4 marks)*

**END OF QUESTIONS**

General Certificate of Education
January 2007
Advanced Level Examination

**MATHEMATICS** **MPC4**
**Unit Pure Core 4**

Thursday 25 January 2007 9.00 am to 10.30 am

**For this paper you must have:**
- an 8-page answer book
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You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**
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- Answer **all** questions.
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**Information**
- The maximum mark for this paper is 75.
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**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

**1** A curve is defined by the parametric equations

$$x = 1 + 2t, \quad y = 1 - 4t^2$$

(a) (i) Find $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$. *(2 marks)*

(ii) Hence find $\dfrac{dy}{dx}$ in terms of $t$. *(2 marks)*

(b) Find an equation of the normal to the curve at the point where $t = 1$. *(4 marks)*

(c) Find a cartesian equation of the curve. *(3 marks)*

**2** The polynomial $f(x)$ is defined by $f(x) = 2x^3 - 7x^2 + 13$.

(a) Use the Remainder Theorem to find the remainder when $f(x)$ is divided by $(2x - 3)$. *(2 marks)*

(b) The polynomial $g(x)$ is defined by $g(x) = 2x^3 - 7x^2 + 13 + d$, where $d$ is a constant.

Given that $(2x - 3)$ is a factor of $g(x)$, show that $d = -4$. *(2 marks)*

(c) Express $g(x)$ in the form $(2x - 3)(x^2 + ax + b)$. *(2 marks)*

**3** (a) Express $\cos 2x$ in terms of $\sin x$. *(1 mark)*

(b) (i) Hence show that $3\sin x - \cos 2x = 2\sin^2 x + 3\sin x - 1$ for all values of $x$. *(2 marks)*

(ii) Solve the equation $3\sin x - \cos 2x = 1$ for $0° < x < 360°$. *(4 marks)*

(c) Use your answer from part (a) to find $\int \sin^2 x \, dx$. *(2 marks)*

**4** (a) (i) Express $\dfrac{3x-5}{x-3}$ in the form $A+\dfrac{B}{x-3}$, where $A$ and $B$ are integers. *(2 marks)*

(ii) Hence find $\displaystyle\int \frac{3x-5}{x-3}\,\mathrm{d}x$. *(2 marks)*

(b) (i) Express $\dfrac{6x-5}{4x^2-25}$ in the form $\dfrac{P}{2x+5}+\dfrac{Q}{2x-5}$, where $P$ and $Q$ are integers. *(3 marks)*

(ii) Hence find $\displaystyle\int \frac{6x-5}{4x^2-25}\,\mathrm{d}x$. *(3 marks)*

**5** (a) Find the binomial expansion of $(1+x)^{\frac{1}{3}}$ up to the term in $x^2$. *(2 marks)*

(b) (i) Show that $(8+3x)^{\frac{1}{3}} \approx 2+\frac{1}{4}x-\frac{1}{32}x^2$ for small values of $x$. *(3 marks)*

(ii) Hence show that $\sqrt[3]{9} \approx \frac{599}{288}$. *(2 marks)*

**6** The points $A$, $B$ and $C$ have coordinates $(3, -2, 4)$, $(5, 4, 0)$ and $(11, 6, -4)$ respectively.

(a) (i) Find the vector $\overrightarrow{BA}$. *(2 marks)*

(ii) Show that the size of angle $ABC$ is $\cos^{-1}\left(-\frac{5}{7}\right)$. *(5 marks)*

(b) The line $l$ has equation $\mathbf{r} = \begin{bmatrix} 8 \\ -3 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$.

(i) Verify that $C$ lies on $l$. *(2 marks)*

(ii) Show that $AB$ is parallel to $l$. *(1 mark)*

(c) The quadrilateral $ABCD$ is a parallelogram. Find the coordinates of $D$. *(3 marks)*

**Turn over for the next question**

**7** (a) Use the identity

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

to express $\tan 2x$ in terms of $\tan x$. *(2 marks)*

(b) Show that

$$2 - 2\tan x - \frac{2\tan x}{\tan 2x} = (1 - \tan x)^2$$

for all values of $x$, $\tan 2x \neq 0$. *(4 marks)*

**8** (a) (i) Solve the differential equation $\frac{\mathrm{d}y}{\mathrm{d}t} = y\sin t$ to obtain $y$ in terms of $t$. *(4 marks)*

(ii) Given that $y = 50$ when $t = \pi$, show that $y = 50\mathrm{e}^{-(1+\cos t)}$. *(3 marks)*

(b) A wave machine at a leisure pool produces waves. The height of the water, $y$ cm, above a fixed point at time $t$ seconds is given by the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y\sin t$$

(i) Given that this height is 50 cm after $\pi$ seconds, find, to the nearest centimetre, the height of the water after 6 seconds. *(2 marks)*

(ii) Find $\frac{\mathrm{d}^2y}{\mathrm{d}t^2}$ and hence verify that the water reaches a maximum height after $\pi$ seconds. *(4 marks)*

**END OF QUESTIONS**

General Certificate of Education
June 2007
Advanced Level Examination

**MATHEMATICS** **MPC4**
**Unit Pure Core 4**

Monday 18 June 2007 9.00 am to 10.30 am

| |
|---|
| **For this paper you must have:**<br>• an 8-page answer book<br>• the **blue** AQA booklet of formulae and statistical tables.<br>You may use a graphics calculator. |

Time allowed: 1 hour 30 minutes

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**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

**1** (a) Find the remainder when $2x^2 + x - 3$ is divided by $2x + 1$. *(2 marks)*

(b) Simplify the algebraic fraction $\dfrac{2x^2 + x - 3}{x^2 - 1}$. *(3 marks)*

**2** (a) (i) Find the binomial expansion of $(1 + x)^{-1}$ up to the term in $x^3$. *(2 marks)*

(ii) Hence, or otherwise, obtain the binomial expansion of $\dfrac{1}{1 + 3x}$ up to the term in $x^3$. *(2 marks)*

(b) Express $\dfrac{1 + 4x}{(1 + x)(1 + 3x)}$ in partial fractions. *(3 marks)*

(c) (i) Find the binomial expansion of $\dfrac{1 + 4x}{(1 + x)(1 + 3x)}$ up to the term in $x^3$. *(3 marks)*

(ii) Find the range of values of $x$ for which the binomial expansion of $\dfrac{1 + 4x}{(1 + x)(1 + 3x)}$ is valid. *(2 marks)*

**3** (a) Express $4\cos x + 3\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 360°$, giving your value for $\alpha$ to the nearest 0.1°. *(3 marks)*

(b) Hence solve the equation $4\cos x + 3\sin x = 2$ in the interval $0° < x < 360°$, giving all solutions to the nearest 0.1°. *(4 marks)*

(c) Write down the minimum value of $4\cos x + 3\sin x$ and find the value of $x$ in the interval $0° < x < 360°$ at which this minimum value occurs. *(3 marks)*

**4** A biologist is researching the growth of a certain species of hamster. She proposes that the length, $x$ cm, of a hamster $t$ days after its birth is given by

$$x = 15 - 12e^{-\frac{t}{14}}$$

(a) Use this model to find:

(i) the length of a hamster when it is born; *(1 mark)*

(ii) the length of a hamster after 14 days, giving your answer to three significant figures. *(2 marks)*

(b) (i) Show that the time for a hamster to grow to 10 cm in length is given by $t = 14\ln\left(\frac{a}{b}\right)$, where $a$ and $b$ are integers. *(3 marks)*

(ii) Find this time to the nearest day. *(1 mark)*

(c) (i) Show that

$$\frac{dx}{dt} = \frac{1}{14}(15 - x)$$ *(3 marks)*

(ii) Find the rate of growth of the hamster, in cm per day, when its length is 8 cm. *(1 mark)*

**5** The point $P(1, a)$, where $a > 0$, lies on the curve $y + 4x = 5x^2y^2$.

(a) Show that $a = 1$. *(2 marks)*

(b) Find the gradient of the curve at $P$. *(7 marks)*

(c) Find an equation of the tangent to the curve at $P$. *(1 mark)*

**Turn over for the next question**

**6** A curve is given by the parametric equations

$$x = \cos\theta \qquad y = \sin 2\theta$$

(a) (i) Find $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}$ and $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}$. *(2 marks)*

(ii) Find the gradient of the curve at the point where $\theta = \dfrac{\pi}{6}$. *(2 marks)*

(b) Show that the cartesian equation of the curve can be written as

$$y^2 = kx^2(1 - x^2)$$

where $k$ is an integer. *(4 marks)*

**7** The lines $l_1$ and $l_2$ have equations $\mathbf{r} = \begin{bmatrix} 8 \\ 6 \\ -9 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ -3 \\ -1 \end{bmatrix}$ and $\mathbf{r} = \begin{bmatrix} -4 \\ 0 \\ 11 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ respectively.

(a) Show that $l_1$ and $l_2$ are perpendicular. *(2 marks)*

(b) Show that $l_1$ and $l_2$ intersect and find the coordinates of the point of intersection, $P$. *(5 marks)*

(c) The point $A(-4, 0, 11)$ lies on $l_2$. The point $B$ on $l_1$ is such that $AP = BP$.

Find the length of $AB$. *(4 marks)*

**8** (a) Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{1 + 2y}}{x^2}$$

given that $y = 4$ when $x = 1$. *(6 marks)*

(b) Show that the solution can be written as $y = \dfrac{1}{2}\left(15 - \dfrac{8}{x} + \dfrac{1}{x^2}\right)$. *(2 marks)*

**END OF QUESTIONS**

General Certificate of Education
January 2008
Advanced Level Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

**MATHEMATICS** **MPC4**
**Unit Pure Core 4**

Thursday 24 January 2008 9.00 am to 10.30 am

| |
|---|
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Time allowed: 1 hour 30 minutes

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**Advice**
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Answer **all** questions.

**1** (a) Given that $\dfrac{3}{9-x^2}$ can be expressed in the form $k\left(\dfrac{1}{3+x}+\dfrac{1}{3-x}\right)$, find the value of the rational number $k$. *(2 marks)*

(b) Show that $\displaystyle\int_1^2 \frac{3}{9-x^2}\,\mathrm{d}x = \frac{1}{2}\ln\left(\frac{a}{b}\right)$, where $a$ and $b$ are integers. *(3 marks)*

**2** (a) The polynomial $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = 2x^3 + 3x^2 - 18x + 8$.

(i) Use the Factor Theorem to show that $(2x-1)$ is a factor of $\mathrm{f}(x)$. *(2 marks)*

(ii) Write $\mathrm{f}(x)$ in the form $(2x-1)(x^2+px+q)$, where $p$ and $q$ are integers. *(2 marks)*

(iii) Simplify the algebraic fraction $\dfrac{4x^2+16x}{2x^3+3x^2-18x+8}$. *(2 marks)*

(b) Express the algebraic fraction $\dfrac{2x^2}{(x+5)(x-3)}$ in the form $A + \dfrac{B+Cx}{(x+5)(x-3)}$, where $A$, $B$ and $C$ are integers. *(4 marks)*

**3** (a) Obtain the binomial expansion of $(1+x)^{\frac{1}{2}}$ up to and including the term in $x^2$. *(2 marks)*

(b) Hence obtain the binomial expansion of $\sqrt{1+\frac{3}{2}x}$ up to and including the term in $x^2$. *(2 marks)*

(c) Hence show that $\sqrt{\dfrac{2+3x}{8}} \approx a + bx + cx^2$ for small values of $x$, where $a$, $b$ and $c$ are constants to be found. *(2 marks)*

**4** David is researching changes in the selling price of houses. One particular house was sold on 1 January 1885 for £20. Sixty years later, on 1 January 1945, it was sold for £2000. David proposes a model

$$P = Ak^t$$

for the selling price, £$P$, of this house, where $t$ is the time in years after 1 January 1885 and $A$ and $k$ are constants.

(a) (i) Write down the value of $A$. *(1 mark)*

(ii) Show that, to six decimal places, $k = 1.079775$. *(2 marks)*

(iii) Use the model, with this value of $k$, to estimate the selling price of this house on 1 January 2008. Give your answer to the nearest £1000. *(2 marks)*

(b) For another house, which was sold for £15 on 1 January 1885, David proposes the model

$$Q = 15 \times 1.082709^t$$

for the selling price, £$Q$, of this house $t$ years after 1 January 1885. Calculate the year in which, according to these models, these two houses would have had the same selling price. *(4 marks)*

**5** A curve is defined by the parametric equations $x = 2t + \dfrac{1}{t^2}$, $y = 2t - \dfrac{1}{t^2}$.

(a) At the point $P$ on the curve, $t = \dfrac{1}{2}$.

(i) Find the coordinates of $P$. *(2 marks)*

(ii) Find an equation of the tangent to the curve at $P$. *(5 marks)*

(b) Show that the cartesian equation of the curve can be written as

$$(x - y)(x + y)^2 = k$$

where $k$ is an integer. *(3 marks)*

**Turn over for the next question**

**6** A curve has equation $3xy - 2y^2 = 4$.

Find the gradient of the curve at the point (2, 1). *(5 marks)*

**7** (a) (i) Express $6\sin\theta + 8\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give your value for $\alpha$ to the nearest 0.1°. *(2 marks)*

(ii) Hence solve the equation $6\sin 2x + 8\cos 2x = 7$, giving all solutions to the nearest 0.1° in the interval $0° < x < 360°$. *(4 marks)*

(b) (i) Prove the identity $\dfrac{\sin 2x}{1 - \cos 2x} = \dfrac{1}{\tan x}$. *(4 marks)*

(ii) Hence solve the equation

$$\frac{\sin 2x}{1 - \cos 2x} = \tan x$$

giving all solutions in the interval $0° < x < 360°$. *(4 marks)*

**8** Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3\cos 3x}{y}$$

given that $y = 2$ when $x = \dfrac{\pi}{2}$. Give your answer in the form $y^2 = \mathrm{f}(x)$. *(5 marks)*

**9** The points $A$ and $B$ lie on the line $l_1$ and have coordinates (2, 5, 1) and (4, 1, −2) respectively.

(a) (i) Find the vector $\overrightarrow{AB}$. *(2 marks)*

(ii) Find a vector equation of the line $l_1$, with parameter $\lambda$. *(1 mark)*

(b) The line $l_2$ has equation $\mathbf{r} = \begin{bmatrix} 1 \\ -3 \\ -1 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$.

(i) Show that the point $P(-2, -3, 5)$ lies on $l_2$. *(2 marks)*

(ii) The point $Q$ lies on $l_1$ and is such that $PQ$ is perpendicular to $l_2$. Find the coordinates of $Q$. *(6 marks)*

**END OF QUESTIONS**

General Certificate of Education
June 2008
Advanced Level Examination

**MATHEMATICS** **MPC4**
**Unit Pure Core 4**

Thursday 12 June 2008 9.00 am to 10.30 am

**For this paper you must have:**
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

**1** The polynomial $f(x)$ is defined by $f(x) = 27x^3 - 9x + 2$.

(a) Find the remainder when $f(x)$ is divided by $3x + 1$. *(2 marks)*

(b) (i) Show that $f\left(-\frac{2}{3}\right) = 0$. *(1 mark)*

(ii) Express $f(x)$ as a product of three linear factors. *(4 marks)*

(iii) Simplify

$$\frac{27x^3 - 9x + 2}{9x^2 + 3x - 2}$$ *(2 marks)*

**2** A curve is defined, for $t \neq 0$, by the parametric equations

$$x = 4t + 3, \quad y = \frac{1}{2t} - 1$$

At the point $P$ on the curve, $t = \frac{1}{2}$.

(a) Find the gradient of the curve at the point $P$. *(4 marks)*

(b) Find an equation of the normal to the curve at the point $P$. *(3 marks)*

(c) Find a cartesian equation of the curve. *(3 marks)*

**3** (a) By writing $\sin 3x$ as $\sin(x + 2x)$, show that $\sin 3x = 3\sin x - 4\sin^3 x$ for all values of $x$. *(5 marks)*

(b) Hence, or otherwise, find $\int \sin^3 x \, dx$. *(3 marks)*

**4** (a) (i) Obtain the binomial expansion of $(1 - x)^{\frac{1}{4}}$ up to and including the term in $x^2$. *(2 marks)*

(ii) Hence show that $(81 - 16x)^{\frac{1}{4}} \approx 3 - \frac{4}{27}x - \frac{8}{729}x^2$ for small values of $x$. *(3 marks)*

(b) Use the result from part (a)(ii) to find an approximation for $\sqrt[4]{80}$, giving your answer to seven decimal places. *(2 marks)*

**5** (a) The angle $\alpha$ is acute and $\sin\alpha = \frac{4}{5}$.

(i) Find the value of $\cos\alpha$. *(1 mark)*

(ii) Express $\cos(\alpha - \beta)$ in terms of $\sin\beta$ and $\cos\beta$. *(2 marks)*

(iii) Given also that the angle $\beta$ is acute and $\cos\beta = \frac{5}{13}$, find the exact value of $\cos(\alpha - \beta)$. *(2 marks)*

(b) (i) Given that $\tan 2x = 1$, show that $\tan^2 x + 2\tan x - 1 = 0$. *(2 marks)*

(ii) Hence, given that $\tan 45° = 1$, show that $\tan 22\frac{1}{2}° = \sqrt{2} - 1$. *(3 marks)*

**6** (a) Express $\dfrac{2}{x^2 - 1}$ in the form $\dfrac{A}{x-1} + \dfrac{B}{x+1}$. *(3 marks)*

(b) Hence find $\displaystyle\int \frac{2}{x^2 - 1}\,\mathrm{d}x$. *(2 marks)*

(c) Solve the differential equation $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2y}{3(x^2 - 1)}$, given that $y = 1$ when $x = 3$.

Show that the solution can be written as $y^3 = \dfrac{2(x-1)}{x+1}$. *(5 marks)*

**7** The coordinates of the points $A$ and $B$ are $(3, -2, 1)$ and $(5, 3, 0)$ respectively.

The line $l$ has equation $\mathbf{r} = \begin{bmatrix} 5 \\ 3 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$.

(a) Find the distance between $A$ and $B$. *(2 marks)*

(b) Find the acute angle between the lines $AB$ and $l$. Give your answer to the nearest degree. *(5 marks)*

(c) The points $B$ and $C$ lie on $l$ such that the distance $AC$ is equal to the distance $AB$. Find the coordinates of $C$. *(5 marks)*

**Turn over for the next question**

**8** (a) The number of fish in a lake is decreasing. After $t$ years, there are $x$ fish in the lake. The rate of decrease of the number of fish is proportional to the number of fish currently in the lake.

(i) Formulate a differential equation, in the variables $x$ and $t$ and a constant of proportionality $k$, where $k > 0$, to model the rate at which the number of fish in the lake is decreasing. *(2 marks)*

(ii) At a certain time, there were 20 000 fish in the lake and the rate of decrease was 500 fish per year. Find the value of $k$. *(2 marks)*

(b) The equation

$$P = 2000 - Ae^{-0.05t}$$

is proposed as a model for the number of fish, $P$, in another lake, where $t$ is the time in years and $A$ is a positive constant.

On 1 January 2008, a biologist estimated that there were 700 fish in this lake.

(i) Taking 1 January 2008 as $t = 0$, find the value of $A$. *(1 mark)*

(ii) Hence find the year during which, according to this model, the number of fish in this lake will first exceed 1900. *(4 marks)*

**END OF QUESTIONS**

General Certificate of Education
January 2009
Advanced Level Examination

**MATHEMATICS** **MPC4**
**Unit Pure Core 4**

Wednesday 21 January 2009 1.30 pm to 3.00 pm

**For this paper you must have:**
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

**1** (a) The polynomial $f(x)$ is defined by $f(x) = 4x^3 - 7x - 3$.

(i) Find $f(-1)$. *(1 mark)*

(ii) Use the Factor Theorem to show that $2x + 1$ is a factor of $f(x)$. *(2 marks)*

(iii) Simplify the algebraic fraction $\dfrac{4x^3 - 7x - 3}{2x^2 + 3x + 1}$. *(3 marks)*

(b) The polynomial $g(x)$ is defined by $g(x) = 4x^3 - 7x + d$. When $g(x)$ is divided by $2x + 1$, the remainder is 2. Find the value of $d$. *(2 marks)*

**2** (a) Express $\sin x - 3\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$. Give your value of $\alpha$ in radians to two decimal places. *(3 marks)*

(b) Hence:

(i) write down the minimum value of $\sin x - 3\cos x$; *(1 mark)*

(ii) find the value of $x$ in the interval $0 < x < 2\pi$ at which this minimum value occurs, giving your value of $x$ in radians to two decimal places. *(2 marks)*

**3** (a) (i) Express $\dfrac{2x + 7}{x + 2}$ in the form $A + \dfrac{B}{x + 2}$, where $A$ and $B$ are integers. *(2 marks)*

(ii) Hence find $\displaystyle\int \frac{2x + 7}{x + 2}\, dx$. *(2 marks)*

(b) (i) Express $\dfrac{28 + 4x^2}{(1 + 3x)(5 - x)^2}$ in the form $\dfrac{P}{1 + 3x} + \dfrac{Q}{5 - x} + \dfrac{R}{(5 - x)^2}$, where $P$, $Q$ and $R$ are constants. *(5 marks)*

(ii) Hence find $\displaystyle\int \frac{28 + 4x^2}{(1 + 3x)(5 - x)^2}\, dx$. *(4 marks)*

**4** (a) (i) Find the binomial expansion of $(1-x)^{\frac{1}{2}}$ up to and including the term in $x^2$. *(2 marks)*

(ii) Hence obtain the binomial expansion of $\sqrt{4-x}$ up to and including the term in $x^2$. *(3 marks)*

(b) Use your answer to part (a)(ii) to find an approximate value for $\sqrt{3}$. Give your answer to three decimal places. *(2 marks)*

**5** (a) Express $\sin 2x$ in terms of $\sin x$ and $\cos x$. *(1 mark)*

(b) Solve the equation

$$5\sin 2x + 3\cos x = 0$$

giving all solutions in the interval $0° \leqslant x \leqslant 360°$ to the nearest 0.1°, where appropriate. *(4 marks)*

(c) Given that $\sin 2x + \cos 2x = 1 + \sin x$ and $\sin x \neq 0$, show that $2(\cos x - \sin x) = 1$. *(4 marks)*

**6** A curve is defined by the equation $x^2y + y^3 = 2x + 1$.

(a) Find the gradient of the curve at the point $(2, 1)$. *(6 marks)*

(b) Show that the $x$-coordinate of any stationary point on this curve satisfies the equation

$$\frac{1}{x^3} = x + 1$$ *(4 marks)*

**Turn over for the next question**

**7** (a) A differential equation is given by $\dfrac{dx}{dt} = -kte^{\frac{1}{2}x}$, where $k$ is a positive constant.

(i) Solve the differential equation. *(3 marks)*

(ii) Hence, given that $x = 6$ when $t = 0$, show that $x = -2\ln\left(\dfrac{kt^2}{4} + e^{-3}\right)$. *(3 marks)*

(b) The population of a colony of insects is decreasing according to the model $\dfrac{dx}{dt} = -kte^{\frac{1}{2}x}$, where $x$ thousands is the number of insects in the colony after time $t$ minutes. Initially, there were 6000 insects in the colony.

Given that $k = 0.004$, find:

(i) the population of the colony after 10 minutes, giving your answer to the nearest hundred; *(2 marks)*

(ii) the time after which there will be no insects left in the colony, giving your answer to the nearest 0.1 of a minute. *(2 marks)*

**8** The points $A$ and $B$ have coordinates $(2, 1, -1)$ and $(3, 1, -2)$ respectively. The angle $OBA$ is $\theta$, where $O$ is the origin.

(a) (i) Find the vector $\overrightarrow{AB}$. *(2 marks)*

(ii) Show that $\cos\theta = \dfrac{5}{2\sqrt{7}}$. *(4 marks)*

(b) The point $C$ is such that $\overrightarrow{OC} = 2\overrightarrow{OB}$. The line $l$ is parallel to $\overrightarrow{AB}$ and passes through the point $C$. Find a vector equation of $l$. *(2 marks)*

(c) The point $D$ lies on $l$ such that angle $ODC = 90°$. Find the coordinates of $D$. *(4 marks)*

**END OF QUESTIONS**

General Certificate of Education
June 2009
Advanced Level Examination

**MATHEMATICS** **MPC4**
**Unit Pure Core 4**

Thursday 11 June 2009 9.00 am to 10.30 am

| |
|---|
| **For this paper you must have:**<br>• an 8-page answer book<br>• the blue AQA booklet of formulae and statistical tables.<br>You may use a graphics calculator. |

Time allowed: 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

**1** (a) Use the Remainder Theorem to find the remainder when $3x^3 + 8x^2 - 3x - 5$ is divided by $3x - 1$. *(2 marks)*

(b) Express $\dfrac{3x^3 + 8x^2 - 3x - 5}{3x - 1}$ in the form $ax^2 + bx + \dfrac{c}{3x - 1}$, where $a$, $b$ and $c$ are integers. *(3 marks)*

**2** A curve is defined by the parametric equations

$$x = \frac{1}{t}, \qquad y = t + \frac{1}{2t}$$

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $t$. *(4 marks)*

(b) Find an equation of the normal to the curve at the point where $t = 1$. *(4 marks)*

(c) Show that the cartesian equation of the curve can be written in the form

$$x^2 - 2xy + k = 0$$

where $k$ is an integer. *(3 marks)*

**3** (a) Find the binomial expansion of $(1 - x)^{-1}$ up to and including the term in $x^2$. *(2 marks)*

(b) (i) Express $\dfrac{3x - 1}{(1 - x)(2 - 3x)}$ in the form $\dfrac{A}{1 - x} + \dfrac{B}{2 - 3x}$, where $A$ and $B$ are integers. *(3 marks)*

(ii) Find the binomial expansion of $\dfrac{3x - 1}{(1 - x)(2 - 3x)}$ up to and including the term in $x^2$. *(6 marks)*

(c) Find the range of values of $x$ for which the binomial expansion of $\dfrac{3x - 1}{(1 - x)(2 - 3x)}$ is valid. *(2 marks)*

**4** A car depreciates in value according to the model

$$V = Ak^t$$

where £$V$ is the value of the car $t$ months from when it was new, and $A$ and $k$ are constants. Its value when new was £12 499 and 36 months later its value was £7000.

(a) (i) Write down the value of $A$. *(1 mark)*

(ii) Show that the value of $k$ is 0.984 025, correct to six decimal places. *(2 marks)*

(b) The value of this car first dropped below £5000 during the $n$th month from new. Find the value of $n$. *(3 marks)*

**5** A curve is defined by the equation $4x^2 + y^2 = 4 + 3xy$.

Find the gradient at the point $(1, 3)$ on this curve. *(5 marks)*

**6** (a) (i) Show that the equation $3\cos 2x + 7\cos x + 5 = 0$ can be written in the form $a\cos^2 x + b\cos x + c = 0$, where $a$, $b$ and $c$ are integers. *(3 marks)*

(ii) Hence find the possible values of $\cos x$. *(2 marks)*

(b) (i) Express $7\sin\theta + 3\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $\alpha$ is an acute angle. Give your value of $\alpha$ to the nearest $0.1°$. *(3 marks)*

(ii) Hence solve the equation $7\sin\theta + 3\cos\theta = 4$ for all solutions in the interval $0° \leqslant \theta \leqslant 360°$, giving $\theta$ to the nearest $0.1°$. *(3 marks)*

(c) (i) Given that $\beta$ is an acute angle and that $\tan\beta = 2\sqrt{2}$, show that $\cos\beta = \frac{1}{3}$. *(2 marks)*

(ii) Hence show that $\sin 2\beta = p\sqrt{2}$, where $p$ is a rational number. *(2 marks)*

**Turn over for the next question**

**7** The points $A$ and $B$ have coordinates $(3, -2, 5)$ and $(4, 0, 1)$ respectively.

The line $l_1$ has equation $\mathbf{r} = \begin{bmatrix} 6 \\ -1 \\ 5 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$.

(a) Find the distance between the points $A$ and $B$. *(2 marks)*

(b) Verify that $B$ lies on $l_1$. *(2 marks)*

(c) The line $l_2$ passes through $A$ and has equation $\mathbf{r} = \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix} + \mu \begin{bmatrix} -1 \\ 3 \\ -8 \end{bmatrix}$.

The lines $l_1$ and $l_2$ intersect at the point $C$. Show that the points $A$, $B$ and $C$ form an isosceles triangle. *(6 marks)*

**8** (a) Solve the differential equation

$$\frac{dx}{dt} = \frac{150 \cos 2t}{x}$$

given that $x = 20$ when $t = \frac{\pi}{4}$, giving your solution in the form $x^2 = f(t)$. *(6 marks)*

(b) The oscillations of a 'baby bouncy cradle' are modelled by the differential equation

$$\frac{dx}{dt} = \frac{150 \cos 2t}{x}$$

where $x$ cm is the height of the cradle above its base $t$ seconds after the cradle begins to oscillate.

Given that the cradle is 20 cm above its base at time $t = \frac{\pi}{4}$ seconds, find:

(i) the height of the cradle above its base 13 seconds after it starts oscillating, giving your answer to the nearest millimetre; *(2 marks)*

(ii) the time at which the cradle will first be 11 cm above its base, giving your answer to the nearest tenth of a second. *(2 marks)*

**END OF QUESTIONS**

General Certificate of Education
Advanced Level Examination
January 2010

# Mathematics MPC4

**Unit Pure Core 4**

**Tuesday 19 January 2010 9.00 am to 10.30 am**

**For this paper you must have:**
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Time allowed**
- 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MPC4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

**1** The polynomial $f(x)$ is defined by $f(x) = 15x^3 + 19x^2 - 4$.

(a) (i) Find $f(-1)$. *(1 mark)*

(ii) Show that $(5x - 2)$ is a factor of $f(x)$. *(2 marks)*

(b) Simplify

$$\frac{15x^2 - 6x}{f(x)}$$

giving your answer in a fully factorised form. *(5 marks)*

**2** (a) Express $\cos x + 3\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your value of $\alpha$, in radians, to three decimal places. *(3 marks)*

(b) (i) Hence write down the minimum value of $\cos x + 3\sin x$. *(1 mark)*

(ii) Find the value of $x$ in the interval $0 \leqslant x \leqslant 2\pi$ at which this minimum occurs, giving your answer, in radians, to three decimal places. *(2 marks)*

(c) Solve the equation $\cos x + 3\sin x = 2$ in the interval $0 \leqslant x \leqslant 2\pi$, giving all solutions, in radians, to three decimal places. *(4 marks)*

**3** (a) (i) Find the binomial expansion of $(1 + x)^{-\frac{1}{3}}$ up to and including the term in $x^2$. *(2 marks)*

(ii) Hence find the binomial expansion of $\left(1 + \frac{3}{4}x\right)^{-\frac{1}{3}}$ up to and including the term in $x^2$. *(2 marks)*

(b) Hence show that $\sqrt[3]{\dfrac{256}{4 + 3x}} \approx a + bx + cx^2$ for small values of $x$, stating the values of the constants $a$, $b$ and $c$. *(3 marks)*

**4** The expression $\dfrac{10x^2+8}{(x+1)(5x-1)}$ can be written in the form $2+\dfrac{A}{x+1}+\dfrac{B}{5x-1}$, where $A$ and $B$ are constants.

(a) Find the values of $A$ and $B$. *(4 marks)*

(b) Hence find $\displaystyle\int \frac{10x^2+8}{(x+1)(5x-1)}\,dx$. *(4 marks)*

**5** A curve is defined by the equation

$$x^2+xy=e^y$$

Find the gradient at the point $(-1, 0)$ on this curve. *(5 marks)*

**6** (a) (i) Express $\sin 2\theta$ and $\cos 2\theta$ in terms of $\sin\theta$ and $\cos\theta$. *(2 marks)*

(ii) Given that $0<\theta<\dfrac{\pi}{2}$ and $\cos\theta=\dfrac{3}{5}$, show that $\sin 2\theta=\dfrac{24}{25}$ and find the value of $\cos 2\theta$. *(2 marks)*

(b) A curve has parametric equations

$$x=3\sin 2\theta,\quad y=4\cos 2\theta$$

(i) Find $\dfrac{dy}{dx}$ in terms of $\theta$. *(3 marks)*

(ii) At the point $P$ on the curve, $\cos\theta=\dfrac{3}{5}$ and $0<\theta<\dfrac{\pi}{2}$. Find an equation of the tangent to the curve at the point $P$. *(3 marks)*

**7** Solve the differential equation $\dfrac{dy}{dx}=\dfrac{1}{y}\cos\left(\dfrac{x}{3}\right)$, given that $y=1$ when $x=\dfrac{\pi}{2}$.

Write your answer in the form $y^2=f(x)$. *(6 marks)*

**Turn over for the next question**

**8** The points $A$, $B$ and $C$ have coordinates $(2, -1, -5)$, $(0, 5, -9)$ and $(9, 2, 3)$ respectively.

The line $l$ has equation $\mathbf{r} = \begin{bmatrix} 2 \\ -1 \\ -5 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$.

(a) Verify that the point $B$ lies on the line $l$. *(2 marks)*

(b) Find the vector $\overrightarrow{BC}$. *(2 marks)*

(c) The point $D$ is such that $\overrightarrow{AD} = 2\overrightarrow{BC}$.

(i) Show that $D$ has coordinates $(20, -7, 19)$. *(2 marks)*

(ii) The point $P$ lies on $l$ where $\lambda = p$. The line $PD$ is perpendicular to $l$. Find the value of $p$. *(5 marks)*

**9** A botanist is investigating the rate of growth of a certain species of toadstool. She observes that a particular toadstool of this type has a height of 57 millimetres at a time 12 hours after it begins to grow.

She proposes the model $h = A\left(1 - e^{-\frac{1}{4}t}\right)$, where $A$ is a constant, for the height $h$ millimetres of the toadstool, $t$ hours after it begins to grow.

(a) Use this model to:

(i) find the height of the toadstool when $t = 0$; *(1 mark)*

(ii) show that $A = 60$, correct to two significant figures. *(2 marks)*

(b) Use the model $h = 60\left(1 - e^{-\frac{1}{4}t}\right)$ to:

(i) show that the time $T$ hours for the toadstool to grow to a height of 48 millimetres is given by

$$T = a \ln b$$

where $a$ and $b$ are integers; *(3 marks)*

(ii) show that $\dfrac{dh}{dt} = 15 - \dfrac{h}{4}$; *(3 marks)*

(iii) find the height of the toadstool when it is growing at a rate of 13 millimetres per hour. *(1 mark)*

**END OF QUESTIONS**

General Certificate of Education
Advanced Level Examination
June 2010

# Mathematics MPC4

**Unit Pure Core 4**

**Tuesday 15 June 2010 9.00 am to 10.30 am**

**For this paper you must have:**
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Time allowed**
- 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

**1 (a)** The polynomial $f(x)$ is defined by $f(x) = 8x^3 + 6x^2 - 14x - 1$.

Find the remainder when $f(x)$ is divided by $(4x - 1)$. *(2 marks)*

**(b)** The polynomial $g(x)$ is defined by $g(x) = 8x^3 + 6x^2 - 14x + d$.

**(i)** Given that $(4x - 1)$ is a factor of $g(x)$, find the value of the constant $d$. *(2 marks)*

**(ii)** Given that $g(x) = (4x - 1)(ax^2 + bx + c)$, find the values of the integers $a$, $b$ and $c$. *(3 marks)*

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**2** A curve is defined by the parametric equations

$$x = 1 - 3t, \qquad y = 1 + 2t^3$$

**(a)** Find $\dfrac{dy}{dx}$ in terms of $t$. *(3 marks)*

**(b)** Find an equation of the normal to the curve at the point where $t = 1$. *(4 marks)*

**(c)** Find a cartesian equation of the curve. *(2 marks)*

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**3 (a) (i)** Express $\dfrac{7x - 3}{(x+1)(3x-2)}$ in the form $\dfrac{A}{x+1} + \dfrac{B}{3x-2}$. *(3 marks)*

**(ii)** Hence find $\displaystyle\int \frac{7x - 3}{(x+1)(3x-2)}\, dx$. *(2 marks)*

**(b)** Express $\dfrac{6x^2 + x + 2}{2x^2 - x + 1}$ in the form $P + \dfrac{Qx + R}{2x^2 - x + 1}$. *(3 marks)*

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**4 (a) (i)** Find the binomial expansion of $(1 + x)^{\frac{3}{2}}$ up to and including the term in $x^2$. *(2 marks)*

**(ii)** Find the binomial expansion of $(16 + 9x)^{\frac{3}{2}}$ up to and including the term in $x^2$. *(3 marks)*

**(b)** Use your answer to part **(a)(ii)** to show that $13^{\frac{3}{2}} \approx 46 + \dfrac{a}{b}$, stating the values of the integers $a$ and $b$. *(2 marks)*

**5 (a) (i)** Show that the equation $3\cos 2x + 2\sin x + 1 = 0$ can be written in the form

$$3\sin^2 x - \sin x - 2 = 0$$ *(3 marks)*

**(ii)** Hence, given that $3\cos 2x + 2\sin x + 1 = 0$, find the possible values of $\sin x$. *(2 marks)*

**(b) (i)** Express $3\cos 2x + 2\sin 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving $\alpha$ to the nearest 0.1°. *(3 marks)*

**(ii)** Hence solve the equation

$$3\cos 2x + 2\sin 2x + 1 = 0$$

for all solutions in the interval $0° < x < 180°$, giving $x$ to the nearest 0.1°. *(3 marks)*

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**6** A curve has equation $x^3 y + \cos(\pi y) = 7$.

**(a)** Find the exact value of the $x$-coordinate at the point on the curve where $y = 1$. *(2 marks)*

**(b)** Find the gradient of the curve at the point where $y = 1$. *(5 marks)*

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**7** The point $A$ has coordinates $(4, -3, 2)$.

The line $l_1$ passes through $A$ and has equation $\mathbf{r} = \begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$.

The line $l_2$ has equation $\mathbf{r} = \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$.

The point $B$ lies on $l_2$ where $\mu = 2$.

**(a)** Find the vector $\overrightarrow{AB}$. *(3 marks)*

**(b) (i)** Show that the lines $l_1$ and $l_2$ intersect. *(4 marks)*

**(ii)** The lines $l_1$ and $l_2$ intersect at the point $P$. Find the coordinates of $P$. *(1 mark)*

**(c)** The point $C$ lies on a line which is parallel to $l_1$ and which passes through the point $B$. The points $A$, $B$, $C$ and $P$ are the vertices of a parallelogram.

Find the coordinates of the two possible positions of the point $C$. *(4 marks)*

**Turn over ▶**

**8 (a)** Solve the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{1}{5}(x+1)^{\frac{1}{2}}$$

given that $x = 80$ when $t = 0$. Give your answer in the form $x = \mathrm{f}(t)$. *(6 marks)*

**(b)** A fungus is spreading on the surface of a wall. The proportion of the wall that is unaffected after time $t$ hours is $x$%. The rate of change of $x$ is modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{1}{5}(x+1)^{\frac{1}{2}}$$

At $t = 0$, the proportion of the wall that is unaffected is 80%. Find the proportion of the wall that will still be unaffected after 60 hours. *(2 marks)*

**(c)** A biologist proposes an alternative model for the rate at which the fungus is spreading on the wall. The total surface area of the wall is $9\,\mathrm{m}^2$. The surface area that is **affected** at time $t$ hours is $A\,\mathrm{m}^2$. The biologist proposes that the rate of change of $A$ is proportional to the product of the surface area that is affected and the surface area that is unaffected.

**(i)** Write down a differential equation for this model.

(You are not required to solve your differential equation.) *(2 marks)*

**(ii)** A solution of the differential equation for this model is given by

$$A = \frac{9}{1 + 4\mathrm{e}^{-0.09t}}$$

Find the time taken for 50% of the area of the wall to be affected. Give your answer in hours to three significant figures. *(4 marks)*

**END OF QUESTIONS**

| Centre Number | | | | | | Candidate Number | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Surname | | | | | | | | | | |
| Other Names | | | | | | | | | | |
| Candidate Signature | | | | | | | | | | |

| For Examiner's Use | |
|---|---|
| Examiner's Initials | |
| Question | Mark |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| TOTAL | |

AQA

General Certificate of Education
Advanced Level Examination
January 2011

# Mathematics MPC4

## Unit Pure Core 4

## Monday 24 January 2011 9.00 am to 10.30 am

**For this paper you must have:**
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

### Time allowed
- 1 hour 30 minutes

### Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

### Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

### Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions in the spaces provided.

**1 (a)** Express $2\sin x + 5\cos x$ in the form $R\sin(x+\alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give your value of $\alpha$ to the nearest $0.1°$. *(3 marks)*

**(b) (i)** Write down the maximum value of $2\sin x + 5\cos x$. *(1 mark)*

**(ii)** Find the value of $x$ in the interval $0° \leqslant x \leqslant 360°$ at which this maximum occurs, giving the value of $x$ to the nearest $0.1°$. *(2 marks)*

QUESTION PART REFERENCE

**2 (a)** The polynomial $f(x)$ is defined by $f(x) = 9x^3 + 18x^2 - x - 2$.

**(i)** Use the Factor Theorem to show that $3x + 1$ is a factor of $f(x)$. *(2 marks)*

**(ii)** Express $f(x)$ as a product of three linear factors. *(3 marks)*

**(iii)** Simplify $\dfrac{9x^3 + 21x^2 + 6x}{f(x)}$. *(3 marks)*

**(b)** When the polynomial $9x^3 + px^2 - x - 2$ is divided by $3x - 2$, the remainder is $-4$.

Find the value of the constant $p$. *(2 marks)*

QUESTION PART REFERENCE

**3 (a)** Express $\dfrac{3+9x}{(1+x)(3+5x)}$ in the form $\dfrac{A}{1+x}+\dfrac{B}{3+5x}$, where $A$ and $B$ are integers.

*(3 marks)*

**(b)** Hence, or otherwise, find the binomial expansion of $\dfrac{3+9x}{(1+x)(3+5x)}$ up to and including the term in $x^2$.

*(7 marks)*

**(c)** Find the range of values of $x$ for which the binomial expansion of $\dfrac{3+9x}{(1+x)(3+5x)}$ is valid.

*(2 marks)*

QUESTION PART REFERENCE

Do not write outside the box

**4** A curve is defined by the parametric equations

$$x = 3e^t, \quad y = e^{2t} - e^{-2t}$$

**(a) (i)** Find the gradient of the curve at the point where $t = 0$. *(3 marks)*

**(ii)** Find an equation of the tangent to the curve at the point where $t = 0$. *(1 mark)*

**(b)** Show that the cartesian equation of the curve can be written in the form

$$y = \frac{x^2}{k} - \frac{k}{x^2}$$

where $k$ is an integer. *(2 marks)*

QUESTION PART REFERENCE

**5** A model for the radioactive decay of a form of iodine is given by

$$m = m_0 \, 2^{-\frac{1}{8}t}$$

The mass of the iodine after $t$ days is $m$ grams. Its initial mass is $m_0$ grams.

**(a)** Use the given model to find the mass that remains after 10 grams of this form of iodine have decayed for 14 days, giving your answer to the nearest gram. *(2 marks)*

**(b)** A mass of $m_0$ grams of this form of iodine decays to $\dfrac{m_0}{16}$ grams in $d$ days.

Find the value of $d$. *(2 marks)*

**(c)** After $n$ days, a mass of this form of iodine has decayed to less than 1% of its initial mass.

Find the minimum integer value of $n$. *(3 marks)*

QUESTION PART REFERENCE

**6 (a) (i)** Given that $\tan 2x + \tan x = 0$, show that $\tan x = 0$ or $\tan^2 x = 3$. *(3 marks)*

**(ii)** Hence find all solutions of $\tan 2x + \tan x = 0$ in the interval $0° < x < 180°$. *(1 mark)*

**(b) (i)** Given that $\cos x \neq 0$, show that the equation

$$\sin 2x = \cos x \cos 2x$$

can be written in the form

$$2\sin^2 x + 2\sin x - 1 = 0$$ *(3 marks)*

**(ii)** Show that all solutions of the equation $2\sin^2 x + 2\sin x - 1 = 0$ are given by $\sin x = \dfrac{\sqrt{3} - 1}{p}$, where $p$ is an integer. *(3 marks)*

QUESTION PART REFERENCE

**7 (a) (i)** Solve the differential equation $\dfrac{dx}{dt} = \sqrt{x}\sin\left(\dfrac{t}{2}\right)$ to find $x$ in terms of $t$. *(3 marks)*

**(ii)** Given that $x = 1$ when $t = 0$, show that the solution can be written as

$$x = (a - \cos bt)^2$$

where $a$ and $b$ are constants to be found. *(3 marks)*

**(b)** The height, $x$ metres, above the ground of a car in a fairground ride at time $t$ seconds is modelled by the differential equation $\dfrac{dx}{dt} = \sqrt{x}\sin\left(\dfrac{t}{2}\right)$.

The car is 1 metre above the ground when $t = 0$.

**(i)** Find the greatest height above the ground reached by the car during the ride. *(2 marks)*

**(ii)** Find the value of $t$ when the car is first 5 metres above the ground, giving your answer to one decimal place. *(2 marks)*

QUESTION PART REFERENCE

**8** The coordinates of the points $A$ and $B$ are $(3,\ -2,\ 4)$ and $(6,\ 0,\ 3)$ respectively.

The line $l_1$ has equation $\mathbf{r} = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$.

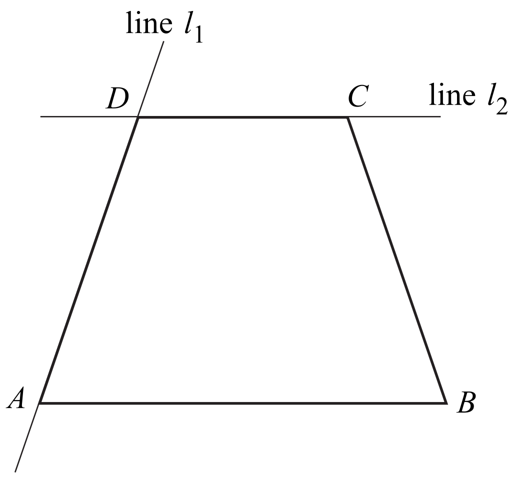
**(a) (i)** Find the vector $\overrightarrow{AB}$. *(2 marks)*

**(ii)** Calculate the acute angle between $\overrightarrow{AB}$ and the line $l_1$, giving your answer to the nearest 0.1°. *(4 marks)*

**(b)** The point $D$ lies on $l_1$ where $\lambda = 2$. The line $l_2$ passes through $D$ and is parallel to $AB$.

**(i)** Find a vector equation of line $l_2$ with parameter $\mu$. *(2 marks)*

**(ii)** The diagram shows a symmetrical trapezium $ABCD$, with angle $DAB$ equal to angle $ABC$.




The point $C$ lies on line $l_2$. The length of $AD$ is equal to the length of $BC$.

Find the coordinates of $C$. *(6 marks)*

QUESTION PART REFERENCE

General Certificate of Education
Advanced Level Examination
June 2011

# Mathematics MPC4

**Unit Pure Core 4**

**Thursday 16 June 2011 1.30 pm to 3.00 pm**

**For this paper you must have:**
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Time allowed**
- 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

**1** The polynomial $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = 4x^3 - 13x + 6$.

**(a)** Find $\mathrm{f}(-2)$. *(1 mark)*

**(b)** Use the Factor Theorem to show that $2x - 3$ is a factor of $\mathrm{f}(x)$. *(2 marks)*

**(c)** Simplify $\dfrac{2x^2 + x - 6}{\mathrm{f}(x)}$. *(4 marks)*

---

**2** The average weekly pay of a footballer at a certain club was £80 on 1 August 1960. By 1 August 1985, this had risen to £2000.

The average weekly pay of a footballer at this club can be modelled by the equation

$$P = Ak^t$$

where £$P$ is the average weekly pay $t$ years after 1 August 1960, and $A$ and $k$ are constants.

**(a) (i)** Write down the value of $A$. *(1 mark)*

**(ii)** Show that the value of $k$ is 1.137411, correct to six decimal places. *(2 marks)*

**(b)** Use this model to predict the year in which, on 1 August, the average weekly pay of a footballer at this club will first exceed £100 000. *(3 marks)*

---

**3 (a) (i)** Find the binomial expansion of $(1 - x)^{\frac{1}{3}}$ up to and including the term in $x^2$. *(2 marks)*

**(ii)** Hence, or otherwise, show that

$$(125 - 27x)^{\frac{1}{3}} \approx 5 + \frac{m}{25}x + \frac{n}{3125}x^2$$

for small values of $x$, stating the values of the integers $m$ and $n$. *(3 marks)*

**(b)** Use your result from part **(a)(ii)** to find an approximate value of $\sqrt[3]{119}$, giving your answer to five decimal places. *(2 marks)*

**4 (a)** A curve is defined by the parametric equations $x = 3\cos 2\theta$, $y = 2\cos\theta$.

**(i)** Show that $\dfrac{dy}{dx} = \dfrac{1}{k\cos\theta}$, where $k$ is an integer. *(4 marks)*

**(ii)** Find an equation of the normal to the curve at the point where $\theta = \dfrac{\pi}{3}$. *(4 marks)*

**(b)** Find the exact value of $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x\, dx$. *(5 marks)*

---

**5** The points $A$ and $B$ have coordinates $(5, 1, -2)$ and $(4, -1, 3)$ respectively.

The line $l$ has equation $\mathbf{r} = \begin{bmatrix} -8 \\ 5 \\ -6 \end{bmatrix} + \mu \begin{bmatrix} 5 \\ 0 \\ -2 \end{bmatrix}$.

**(a)** Find a vector equation of the line that passes through $A$ and $B$. *(3 marks)*

**(b) (i)** Show that the line that passes through $A$ and $B$ intersects the line $l$, and find the coordinates of the point of intersection, $P$. *(4 marks)*

**(ii)** The point $C$ lies on $l$ such that triangle $PBC$ has a right angle at $B$. Find the coordinates of $C$. *(5 marks)*

---

**6** A curve is defined by the equation $2y + e^{2x}y^2 = x^2 + C$, where $C$ is a constant. The point $P\left(1, \dfrac{1}{e}\right)$ lies on the curve.

**(a)** Find the exact value of $C$. *(1 mark)*

**(b)** Find an expression for $\dfrac{dy}{dx}$ in terms of $x$ and $y$. *(7 marks)*

**(c)** Verify that $P\left(1, \dfrac{1}{e}\right)$ is a stationary point on the curve. *(2 marks)*

Turn over ▶

**7** A giant snowball is melting. The snowball can be modelled as a sphere whose surface area is decreasing at a constant rate with respect to time. The surface area of the sphere is $A\,\text{cm}^2$ at time $t$ days after it begins to melt.

**(a)** Write down a differential equation in terms of the variables $A$ and $t$ and a constant $k$, where $k > 0$, to model the melting snowball. *(2 marks)*

**(b) (i)** Initially, the radius of the snowball is 60 cm, and 9 days later, the radius has halved.

Show that $A = 1200\pi(12 - t)$.

(You may assume that the surface area of a sphere is given by $A = 4\pi r^2$, where $r$ is the radius.) *(4 marks)*

**(ii)** Use this model to find the number of days that it takes the snowball to melt completely. *(1 mark)*

---

**8 (a)** Express $\dfrac{1}{(3-2x)(1-x)^2}$ in the form $\dfrac{A}{3-2x} + \dfrac{B}{1-x} + \dfrac{C}{(1-x)^2}$. *(4 marks)*

**(b)** Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\sqrt{y}}{(3-2x)(1-x)^2}$$

where $y = 0$ when $x = 0$, expressing your answer in the form

$$y^p = q\ln[\mathrm{f}(x)] + \frac{x}{1-x}$$

where $p$ and $q$ are constants. *(9 marks)*

**END OF QUESTIONS**

General Certificate of Education
Advanced Level Examination
January 2012

# Mathematics MPC4

**Unit Pure Core 4**

**Monday 23 January 2012 9.00 am to 10.30 am**

**For this paper you must have:**
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Time allowed**
- 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

**1 (a)** Express $\dfrac{2x+3}{4x^2-1}$ in the form $\dfrac{A}{2x-1}+\dfrac{B}{2x+1}$, where $A$ and $B$ are integers. *(3 marks)*

**(b)** Express $\dfrac{12x^3-7x-6}{4x^2-1}$ in the form $Cx+\dfrac{D(2x+3)}{4x^2-1}$, where $C$ and $D$ are integers. *(3 marks)*

**(c)** Evaluate $\displaystyle\int_1^2 \frac{12x^3-7x-6}{4x^2-1}\,dx$, giving your answer in the form $p+\ln q$, where $p$ and $q$ are rational numbers. *(5 marks)*

---

**2** Angle $\alpha$ is acute and $\cos\alpha=\frac{3}{5}$. Angle $\beta$ is **obtuse** and $\sin\beta=\frac{1}{2}$.

**(a) (i)** Find the value of $\tan\alpha$ as a fraction. *(1 mark)*

**(ii)** Find the value of $\tan\beta$ in surd form. *(2 marks)*

**(b)** Hence show that $\tan(\alpha+\beta)=\dfrac{m\sqrt{3}-n}{n\sqrt{3}+m}$, where $m$ and $n$ are integers. *(3 marks)*

---

**3 (a)** Find the binomial expansion of $(1+6x)^{\frac{2}{3}}$ up to and including the term in $x^2$. *(2 marks)*

**(b)** Find the binomial expansion of $(8+6x)^{\frac{2}{3}}$ up to and including the term in $x^2$. *(3 marks)*

**(c)** Use your answer from part **(b)** to find an estimate for $\sqrt[3]{100}$ in the form $\dfrac{a}{b}$, where $a$ and $b$ are integers. *(2 marks)*

**4** A scientist is testing models for the growth and decay of colonies of bacteria.

For a particular colony, which is growing, the model is $P = Ae^{\frac{1}{8}t}$, where $P$ is the number of bacteria after a time $t$ minutes and $A$ is a constant.

**(a)** This growing colony consists initially of 500 bacteria. Calculate the number of bacteria, according to the model, after one hour. Give your answer to the nearest thousand. *(2 marks)*

**(b)** For a second colony, which is decaying, the model is $Q = 500\,000e^{-\frac{1}{8}t}$, where $Q$ is the number of bacteria after a time $t$ minutes.

Initially, the growing colony has 500 bacteria and, at the same time, the decaying colony has 500 000 bacteria.

**(i)** Find the time at which the populations of the two colonies will be equal, giving your answer to the nearest 0.1 of a minute. *(3 marks)*

**(ii)** The population of the growing colony will exceed that of the decaying colony by 45 000 bacteria at time $T$ minutes.

Show that

$$\left(e^{\frac{1}{8}T}\right)^2 - 90e^{\frac{1}{8}T} - 1000 = 0$$

and hence find the value of $T$, giving your answer to one decimal place. *(4 marks)*

---

**5** A curve is defined by the parametric equations

$$x = 8t^2 - t\,, \qquad y = \frac{3}{t}$$

**(a)** Show that the cartesian equation of the curve can be written as $xy^2 + 3y = k$, stating the value of the integer $k$. *(2 marks)*

**(b) (i)** Find an equation of the tangent to the curve at the point $P$, where $t = \frac{1}{4}$. *(7 marks)*

**(ii)** Verify that the tangent at $P$ intersects the curve when $x = \frac{3}{2}$. *(2 marks)*

**Turn over ▶**

**6 (a)** Use the Factor Theorem to show that $4x - 3$ is a factor of

$$16x^3 + 11x - 15$$ *(2 marks)*

**(b)** Given that $x = \cos\theta$, show that the equation

$$27\cos\theta\cos 2\theta + 19\sin\theta\sin 2\theta - 15 = 0$$

can be written in the form

$$16x^3 + 11x - 15 = 0$$ *(4 marks)*

**(c)** Hence show that the only solutions of the equation

$$27\cos\theta\cos 2\theta + 19\sin\theta\sin 2\theta - 15 = 0$$

are given by $\cos\theta = \frac{3}{4}$. *(4 marks)*

---

**7** Solve the differential equation

$$\frac{dy}{dx} = y^2 x \sin 3x$$

given that $y = 1$ when $x = \frac{\pi}{6}$. Give your answer in the form $y = \frac{9}{f(x)}$. *(9 marks)*

---

**8** The points $A$ and $B$ have coordinates $(4, -2, 3)$ and $(2, 0, -1)$ respectively.

The line $l$ passes through $A$ and has equation $\mathbf{r} = \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix}$.

**(a) (i)** Find the vector $\overrightarrow{AB}$. *(2 marks)*

**(ii)** Find the acute angle between $AB$ and the line $l$, giving your answer to the nearest degree. *(4 marks)*

**(b)** The point $C$ lies on the line $l$ such that the angle $ABC$ is a right angle. Given that $ABCD$ is a rectangle, find the coordinates of the point $D$. *(6 marks)*

General Certificate of Education
Advanced Level Examination
June 2012

# Mathematics MPC4

**Unit Pure Core 4**

**Thursday 14 June 2012 9.00 am to 10.30 am**

**For this paper you must have:**
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Time allowed**
- 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

**1 (a) (i)** Express $\dfrac{5x-6}{x(x-3)}$ in the form $\dfrac{A}{x}+\dfrac{B}{x-3}$. *(2 marks)*

**(ii)** Find $\displaystyle\int \frac{5x-6}{x(x-3)}\,\mathrm{d}x$. *(2 marks)*

**(b) (i)** Given that

$$4x^3+5x-2=(2x+1)(2x^2+px+q)+r$$

find the values of the constants $p$, $q$ and $r$. *(4 marks)*

**(ii)** Find $\displaystyle\int \frac{4x^3+5x-2}{2x+1}\,\mathrm{d}x$. *(3 marks)*

---

**2 (a)** Express $\sin x - 3\cos x$ in the form $R\sin(x-\alpha)$, where $R>0$ and $0° < \alpha < 90°$, giving your value of $\alpha$ to the nearest $0.1°$. *(3 marks)*

**(b)** Hence find the values of $x$ in the interval $0° < x < 360°$ for which

$$\sin x - 3\cos x + 2 = 0$$

giving your values of $x$ to the nearest degree. *(4 marks)*

---

**3 (a)** Find the binomial expansion of $(1+4x)^{\frac{1}{2}}$ up to and including the term in $x^2$. *(2 marks)*

**(b) (i)** Find the binomial expansion of $(4-x)^{-\frac{1}{2}}$ up to and including the term in $x^2$. *(3 marks)*

**(ii)** State the range of values of $x$ for which the expansion in part **(b)(i)** is valid. *(1 mark)*

**(c)** Find the binomial expansion of $\sqrt{\dfrac{1+4x}{4-x}}$ up to and including the term in $x^2$. *(2 marks)*

**4** The value, £$V$, of an initial investment, £$P$, at the end of $n$ years is given by the formula

$$V = P\left(1 + \frac{r}{100}\right)^n$$

where $r$% per year is the fixed interest rate.

Mr Brown invests £1000 in Barcelona Bank at a fixed interest rate of 3% per year.

**(a) (i)** Find the value of Mr Brown's investment at the end of 5 years. Give your value to the nearest £10. *(1 mark)*

**(ii)** The value of Mr Brown's investment will first exceed £2000 after $N$ complete years.

Find the value of $N$. *(3 marks)*

**(b)** Mrs White invests £1500 in Bilbao Bank at a fixed interest rate of 1.5% per year. Mr Brown and Mrs White invest their money at the same time. The value of Mr Brown's investment will first exceed the value of Mrs White's investment after $T$ complete years.

Find the value of $T$. *(4 marks)*

---

**5** A curve is defined by the parametric equations

$$x = 2\cos\theta\,, \qquad y = 3\sin 2\theta$$

**(a) (i)** Show that

$$\frac{dy}{dx} = a\sin\theta + b\operatorname{cosec}\theta$$

where $a$ and $b$ are integers. *(4 marks)*

**(ii)** Find the gradient of the normal to the curve at the point where $\theta = \frac{\pi}{6}$. *(2 marks)*

**(b)** Show that the cartesian equation of the curve can be expressed as

$$y^2 = px^2(4 - x^2)$$

where $p$ is a rational number. *(3 marks)*

---

**6** A curve is defined by the equation $9x^2 - 6xy + 4y^2 = 3$.

Find the coordinates of the two stationary points of this curve. *(8 marks)*

**Turn over ▶**

**7** The line $l_1$ has equation $\mathbf{r} = \begin{bmatrix} 0 \\ -2 \\ q \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$, where $q$ is an integer.

The line $l_2$ has equation $\mathbf{r} = \begin{bmatrix} 8 \\ 3 \\ 5 \end{bmatrix} + \mu \begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix}$.

The lines $l_1$ and $l_2$ intersect at the point $P$.

**(a)** Show that $q = 4$ and find the coordinates of $P$. *(3 marks)*

**(b)** Show that $l_1$ and $l_2$ are perpendicular. *(1 mark)*

**(c)** The point $A$ lies on the line $l_1$ where $\lambda = 1$.

**(i)** Find $AP^2$. *(2 marks)*

**(ii)** The point $B$ lies on the line $l_2$ so that the right-angled triangle $APB$ is isosceles.

Find the coordinates of the two possible positions of $B$. *(6 marks)*

---

**8 (a)** A water tank has a height of 2 metres. The depth of the water in the tank is $h$ metres at time $t$ minutes after water begins to enter the tank. The rate at which the depth of the water in the tank increases is proportional to the difference between the height of the tank and the depth of the water.

Write down a differential equation in the variables $h$ and $t$ and a positive constant $k$.

(You are not required to solve your differential equation.) *(3 marks)*

**(b) (i)** Another water tank is filling in such a way that $t$ minutes after the water is turned on, the depth of the water, $x$ metres, increases according to the differential equation

$$\frac{dx}{dt} = \frac{1}{15x\sqrt{2x-1}}$$

The depth of the water is 1 metre when the water is first turned on.

Solve this differential equation to find $t$ as a function of $x$. *(8 marks)*

**(ii)** Calculate the time taken for the depth of the water in the tank to reach 2 metres, giving your answer to the nearest 0.1 of a minute. *(1 mark)*

General Certificate of Education
Advanced Level Examination
January 2013

# Mathematics MPC4

**Unit Pure Core 4**

**Friday 25 January 2013 1.30 pm to 3.00 pm**

**For this paper you must have:**
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Time allowed**
- 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

**1** The polynomial $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = 2x^3 + x^2 - 8x - 7$.

**(a)** Use the Remainder Theorem to find the remainder when $\mathrm{f}(x)$ is divided by $(2x + 1)$. *(2 marks)*

**(b)** The polynomial $\mathrm{g}(x)$ is defined by $\mathrm{g}(x) = \mathrm{f}(x) + d$, where $d$ is a constant.

**(i)** Given that $(2x + 1)$ is a factor of $\mathrm{g}(x)$, show that $\mathrm{g}(x) = 2x^3 + x^2 - 8x - 4$. *(1 mark)*

**(ii)** Given that $\mathrm{g}(x)$ can be written as $\mathrm{g}(x) = (2x + 1)(x^2 + a)$, where $a$ is an integer, express $\mathrm{g}(x)$ as a product of three linear factors. *(1 mark)*

**(iii)** Hence, or otherwise, show that $\dfrac{\mathrm{g}(x)}{2x^3 - 3x^2 - 2x} = p + \dfrac{q}{x}$, where $p$ and $q$ are integers. *(3 marks)*

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**2** It is given that $\mathrm{f}(x) = \dfrac{7x - 1}{(1 + 3x)(3 - x)}$.

**(a)** Express $\mathrm{f}(x)$ in the form $\dfrac{A}{3 - x} + \dfrac{B}{1 + 3x}$, where $A$ and $B$ are integers. *(3 marks)*

**(b) (i)** Find the first three terms of the binomial expansion of $\mathrm{f}(x)$ in the form $a + bx + cx^2$, where $a$, $b$ and $c$ are rational numbers. *(7 marks)*

**(ii)** State why the binomial expansion cannot be expected to give a good approximation to $\mathrm{f}(x)$ at $x = 0.4$. *(1 mark)*

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**3 (a) (i)** Express $3\cos x + 2\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving your value of $\alpha$ to the nearest $0.1°$. *(3 marks)*

**(ii)** Hence find the minimum value of $3\cos x + 2\sin x$ and the value of $x$ in the interval $0° < x < 360°$ where the minimum occurs. Give your value of $x$ to the nearest $0.1°$. *(3 marks)*

**(b) (i)** Show that $\cot x - \sin 2x = \cot x \cos 2x$ for $0° < x < 180°$. *(3 marks)*

**(ii)** Hence, or otherwise, solve the equation

$$\cot x - \sin 2x = 0$$

in the interval $0° < x < 180°$. *(3 marks)*

**4 (a)** A curve is defined by the equation $x^2 - y^2 = 8$.

**(i)** Show that at any point $(p,\ q)$ on the curve, where $q \neq 0$, the gradient of the curve is given by $\dfrac{dy}{dx} = \dfrac{p}{q}$. *(2 marks)*

**(ii)** Show that the tangents at the points $(p,\ q)$ and $(p,\ -q)$ intersect on the $x$-axis. *(4 marks)*

**(b)** Show that $x = t + \dfrac{2}{t}$, $y = t - \dfrac{2}{t}$ are parametric equations of the curve $x^2 - y^2 = 8$. *(2 marks)*

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**5 (a)** Find $\displaystyle\int x\sqrt{x^2 + 3}\,dx$. *(2 marks)*

**(b)** Solve the differential equation

$$\frac{dy}{dx} = \frac{x\sqrt{x^2 + 3}}{e^{2y}}$$

given that $y = 0$ when $x = 1$. Give your answer in the form $y = f(x)$. *(7 marks)*

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**6 (a)** The points $A$, $B$ and $C$ have coordinates $(3,\ 1,\ -6)$, $(5,\ -2,\ 0)$ and $(8,\ -4,\ -6)$ respectively.

**(i)** Show that the vector $\overrightarrow{AC}$ is given by $\overrightarrow{AC} = n\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, where $n$ is an integer. *(1 mark)*

**(ii)** Show that the acute angle $ACB$ is given by $\cos^{-1}\left(\dfrac{5\sqrt{2}}{14}\right)$. *(4 marks)*

**(b)** Find a vector equation of the line $AC$. *(2 marks)*

**(c)** The point $D$ has coordinates $(6,\ -1,\ p)$. It is given that the lines $AC$ and $BD$ intersect.

**(i)** Find the value of $p$. *(4 marks)*

**(ii)** Show that $ABCD$ is a rhombus, and state the length of each of its sides. *(4 marks)*

**Turn over ▶**

**7** A biologist is investigating the growth of a population of a species of rodent. The biologist proposes the model

$$N = \frac{500}{1 + 9e^{-\frac{t}{8}}}$$

for the number of rodents, $N$, in the population $t$ weeks after the start of the investigation.

Use this model to answer the following questions.

**(a) (i)** Find the size of the population at the start of the investigation. *(1 mark)*

**(ii)** Find the size of the population 24 weeks after the start of the investigation. Give your answer to the nearest whole number. *(1 mark)*

**(iii)** Find the number of weeks that it will take the population to reach 400. Give your answer in the form $t = r \ln s$, where $r$ and $s$ are integers. *(3 marks)*

**(b) (i)** Show that the rate of growth, $\frac{dN}{dt}$, is given by

$$\frac{dN}{dt} = \frac{N}{4000}(500 - N)$$ *(4 marks)*

**(ii)** The maximum rate of growth occurs after $T$ weeks. Find the value of $T$. *(4 marks)*

General Certificate of Education
Advanced Level Examination
June 2013

# Mathematics MPC4

**Unit Pure Core 4**

**Monday 10 June 2013 9.00 am to 10.30 am**

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You may use a graphics calculator.

**Time allowed**
- 1 hour 30 minutes

**Instructions**
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- Do all rough work in this book. Cross through any work that you do not want to be marked.

**Information**
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- The maximum mark for this paper is 75.

**Advice**
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**1 (a) (i)** Express $\dfrac{5-8x}{(2+x)(1-3x)}$ in the form $\dfrac{A}{2+x}+\dfrac{B}{1-3x}$, where $A$ and $B$ are integers. *(3 marks)*

**(ii)** Hence show that $\displaystyle\int_{-1}^{0}\frac{5-8x}{(2+x)(1-3x)}\,\mathrm{d}x = p\ln 2$, where $p$ is rational. *(4 marks)*

**(b) (i)** Given that $\dfrac{9-18x-6x^2}{2-5x-3x^2}$ can be written as $C+\dfrac{5-8x}{2-5x-3x^2}$, find the value of $C$. *(1 mark)*

**(ii)** Hence find the exact value of the area of the region bounded by the curve $y=\dfrac{9-18x-6x^2}{2-5x-3x^2}$, the $x$-axis and the lines $x=-1$ and $x=0$.

You may assume that $y>0$ when $-1\leqslant x\leqslant 0$. *(2 marks)*

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**2** The acute angles $\alpha$ and $\beta$ are given by $\tan\alpha=\dfrac{2}{\sqrt{5}}$ and $\tan\beta=\frac{1}{2}$.

**(a) (i)** Show that $\sin\alpha=\frac{2}{3}$, and find the exact value of $\cos\alpha$. *(2 marks)*

**(ii)** Hence find the exact value of $\sin 2\alpha$. *(2 marks)*

**(b)** Show that the exact value of $\cos(\alpha-\beta)$ can be expressed as $\frac{2}{15}(k+\sqrt{5})$, where $k$ is an integer. *(4 marks)*

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**3 (a)** Find the binomial expansion of $(1+6x)^{-\frac{1}{3}}$ up to and including the term in $x^2$. *(2 marks)*

**(b) (i)** Find the binomial expansion of $(27+6x)^{-\frac{1}{3}}$ up to and including the term in $x^2$, simplifying the coefficients. *(3 marks)*

**(ii)** Given that $\sqrt[3]{\dfrac{2}{7}}=\dfrac{2}{\sqrt[3]{28}}$, use your binomial expansion from part **(b)(i)** to obtain an approximation to $\sqrt[3]{\dfrac{2}{7}}$, giving your answer to six decimal places. *(2 marks)*

**4** A curve is defined by the parametric equations $x = 8e^{-2t} - 4$, $y = 2e^{2t} + 4$.

**(a)** Find $\dfrac{dy}{dx}$ in terms of $t$. *(3 marks)*

**(b)** The point $P$, where $t = \ln 2$, lies on the curve.

**(i)** Find the gradient of the curve at $P$. *(1 mark)*

**(ii)** Find the coordinates of $P$. *(2 marks)*

**(iii)** The normal at $P$ crosses the $x$-axis at the point $Q$. Find the coordinates of $Q$. *(3 marks)*

**(c)** Find the Cartesian equation of the curve in the form $xy + 4y - 4x = k$, where $k$ is an integer. *(3 marks)*

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**5** The polynomial $f(x)$ is defined by $f(x) = 4x^3 - 11x - 3$.

**(a)** Use the Factor Theorem to show that $(2x + 3)$ is a factor of $f(x)$. *(2 marks)*

**(b)** Write $f(x)$ in the form $(2x + 3)(ax^2 + bx + c)$, where $a$, $b$ and $c$ are integers. *(2 marks)*

**(c) (i)** Show that the equation $2\cos 2\theta \sin\theta + 9\sin\theta + 3 = 0$ can be written as $4x^3 - 11x - 3 = 0$, where $x = \sin\theta$. *(3 marks)*

**(ii)** Hence find all solutions of the equation $2\cos 2\theta \sin\theta + 9\sin\theta + 3 = 0$ in the interval $0° < \theta < 360°$, giving your solutions to the nearest degree. *(4 marks)*

**6** The points $A$, $B$ and $C$ have coordinates $(3, -2, 4)$, $(1, -5, 6)$ and $(-4, 5, -1)$ respectively.

The line $l$ passes through $A$ and has equation $\mathbf{r} = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} 7 \\ -7 \\ 5 \end{bmatrix}$.

**(a)** Show that the point $C$ lies on the line $l$. *(2 marks)*

**(b)** Find a vector equation of the line that passes through points $A$ and $B$. *(3 marks)*

**(c)** The point $D$ lies on the line through $A$ and $B$ such that the angle $CDA$ is a right angle.

Find the coordinates of $D$. *(5 marks)*

**(d)** The point $E$ lies on the line through $A$ and $B$ such that the area of triangle $ACE$ is three times the area of triangle $ACD$.

Find the coordinates of the two possible positions of $E$. *(4 marks)*

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**7** The height of the tide in a certain harbour is $h$ metres at time $t$ hours. Successive high tides occur every 12 hours.

The **rate of change** of the height of the tide can be modelled by a function of the form $a\cos(kt)$, where $a$ and $k$ are constants. The largest value of this rate of change is 1.3 metres per hour.

Write down a differential equation in the variables $h$ and $t$. State the values of the constants $a$ and $k$. *(3 marks)*

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**8 (a)** Find $\int t\cos\left(\frac{\pi}{4}t\right) \mathrm{d}t$. *(4 marks)*

**(b)** The platform of a theme park ride oscillates vertically. For the first 75 seconds of the ride,

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{t\cos\left(\frac{\pi}{4}t\right)}{32x}$$

where $x$ metres is the height of the platform above the ground after time $t$ seconds.

At $t = 0$, the height of the platform above the ground is 4 metres.

Find the height of the platform after 45 seconds, giving your answer to the nearest centimetre. *(6 marks)*